

Waves & Optics
Phy 211 Course Text, Exercises, and Laboratories

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Contents

I	Geometric Optics	11
1	The Rectilinear Propagation of Light	13
1.1	A Demonstration of Shadows	13
1.2	Huygens' Principle	14
1.2.1	Wave fronts	14
1.2.2	The Law of Rectilinear Propagation	15
1.2.3	Physical Evidence	16
1.2.4	Light Rays and the Anatomy of a Shadow	16
1.3	Exercises	20
2	Reflection and Refraction	21
2.1	A Demonstration of Reflection	21
2.2	Fermat's Principle	22
2.3	The Path of a Reflected Light Ray	22
2.4	A Demonstration of Refraction	23
2.5	The Path of a Refracted Light Ray	24
2.6	Total Internal Reflection	26
2.6.1	Demonstrating Light Traveling from Glass to Air	26
2.6.2	The Critical Angle	26
2.6.3	Example # 1: The Surface of the Water	27
2.6.4	Example # 2: Fiber Optic Cables	28
2.7	Light Traveling in a Medium	29
2.7.1	Actual vs. Apparent Distances	29
2.7.2	The Optical Path Length	30
2.7.3	Apparent Source Distance	31
2.7.4	Background Information: Wavelengths of Light	32
2.8	Prisms and Dispersion	33
2.8.1	Index of Refraction's Dependence on Wavelength	33
2.8.2	The Prism Equation	34
2.8.3	Example # 1: Creating a Rainbow	36
2.9	The Index of Refraction in Air	37
2.9.1	Dependence on Air Density: Positions of Stars	37
2.9.2	Dependence on Air Temperature: Mirages	38
2.9.3	Example # 1: The Desert Mirage	38

2.9.4	Example # 2: The Flying Dutchman	39
2.10	A Remarkable Fact	39
2.11	Exercises	41
3	Spherical Mirrors	43
3.1	Demonstrating Reflection from a (Concave) Spherical Surface . .	43
3.2	The Mirror Equation	44
3.3	Magnification	45
3.4	Light Ray Diagrams	47
3.5	Flat Mirrors	48
3.6	Convex Mirrors	49
3.6.1	The Mirror Equation: Sign Conventions	49
3.6.2	Ray Diagrams	50
3.7	Exercises	51
4	Spherical Lenses	53
4.1	Refraction at a Curved Surface	53
4.2	Refraction Through Two Curved Surfaces: Lenses	56
4.3	Thin Lenses	57
4.3.1	Light Ray Diagrams for Lenses	57
4.3.2	Some Vocabulary	57
4.4	Exercises	60
5	Applications of Geometric Optics	63
5.1	Overview	63
5.2	The Human Eye	63
5.3	Corrective Lenses	66
5.3.1	Correcting for Far-sightedness	66
5.3.2	Correcting for Near-sightedness	66
5.4	Combinations of Lenses	67
5.5	Telescopes	70
5.6	Microscopes	72
5.7	Glass Filters	73
5.8	Optical Aberrations	74
5.8.1	Chromatic Aberration	74
5.8.2	Spherical Aberration	76
5.8.3	Coma	76
5.8.4	Astigmatism	77
5.8.5	Field Curvature	77
5.8.6	Distortion	77
5.9	Exercises	79

II	Oscillations & Waves	81
6	Simple Harmonic Motion	83
6.1	Demonstrating Simple Harmonic Motion with a Mass on a String	83
6.2	Position as a Function of Time	84
6.3	Deriving Equations for Velocity and Acceleration	84
6.4	Example # 1: Frictionless Mass on a Spring	85
6.4.1	Motion and Frequency of the Mass on a Spring	86
6.4.2	Mechanical Energy of the Mass on a Spring	87
6.5	Example # 2: Vertical Mass on a Spring	88
6.6	Example # 3: Simple Pendulum	88
6.7	Example # 4: Large Amplitude Pendula	90
6.8	Example # 5: Torsional Oscillator	91
6.9	Example # 6: A Physical Pendulum	93
6.10	Exercises	95
7	Damped Oscillations	97
7.1	Demonstrating Harmonic Oscillation with Friction	97
7.2	A Single Damping Force	98
7.2.1	Deriving the Equation of Motion	98
7.2.2	Under Damping	100
7.2.3	Critical Damping	101
7.2.4	Over Damping	102
7.2.5	Energy of a (Weakly-)Damped Harmonic Oscillator	102
7.3	Examples # 1 & # 2: Car Shocks and Screen Doors	105
7.4	Example # 3: The Building in the Earthquake	105
7.5	The Quality Factor	106
7.6	Resonance for Cases of Weak Damping	107
7.7	Driving a Damped Oscillator	107
7.7.1	Deriving the Equation of Motion	108
7.7.2	The Amplitude and Phase Shift for Any Frequency	109
7.7.3	Driving at the Resonant Frequency	109
7.8	Example # 4: The Bridge in the Wind	110
7.9	Exercises	111
8	Wave Motion	113
8.1	Demonstrating Transverse and Longitudinal Waves	113
8.2	Wave Pulse on a String	114
8.3	The Speed of a Wave Pulse	115
8.3.1	Derivation for the Pulse on a String	115
8.3.2	Speed in a Fluid	117
8.3.3	Speed in an Ideal Gas	118
8.4	The Wave Equation	119
8.4.1	Generally Speaking	119
8.4.2	The Wave Equation for Waves on a String and in Fluids	120
8.5	Exercises	121

9	Harmonic Waves	123
9.1	A Type of Periodic Wave	123
9.2	The Equations of Motion for Harmonic Waves	124
9.3	The Motion of a Fixed Point Undergoing Harmonic Motion	125
9.4	Power of a Wave on a String	126
9.5	Harmonic Sound Waves	128
9.5.1	Displacement of Air Molecules	128
9.5.2	Pressure in the Sound Wave	130
9.5.3	Energy in Sound Waves	131
9.5.4	Intensity of Sound Waves	132
9.5.5	Measuring Sound Intensity	133
9.6	Exercises	135
10	Phenomena of Traveling Waves	137
10.1	The Doppler Effect for Sound Waves	137
10.1.1	Deriving an Equation for the Frequency Heard	137
10.1.2	Supersonic Motion and Shock Waves	140
10.2	Traveling Waves and Barriers	141
10.2.1	Demonstrating Traveling Wave Reflection	141
10.2.2	Reflection and Transmission	142
10.2.3	True for All Traveling Waves	143
10.2.4	Diffraction of Traveling Waves	144
10.3	Exercises	145
11	The Superposition of Multiple Waves	147
11.1	The Superposition Principle	147
11.2	Harmonic Wave Interference	147
11.2.1	Phase Shift due to Path Length Difference	149
11.3	Beat Frequencies	150
11.4	Standing Waves on a String	150
11.4.1	Wave Functions for Standing Waves on a String	152
11.4.2	Example # 1: Stringed Instruments	152
11.5	Standing Waves in a Pipe	153
11.5.1	Example # 2: Pipe Organ	155
11.5.2	Example # 3: Brass and Woodwind Instruments	156
11.6	The Superposition of Standing Waves	156
11.6.1	Sum of All the Waves	156
11.7	Exercises	159
III	Wave Optics	161
12	The Wave Equation for Light	163
12.1	Statement of the Problem	163
12.2	A Review of Maxwell's Equations	163
12.3	Relating Space and Time Derivatives	165

12.3.1	With Faraday's Law	165
12.3.2	With Ampère's Law	166
12.3.3	One Equation, in Terms of the Electric Field Only	168
12.3.4	The Speed of Light, Defined	168
12.3.5	Relating the Electric and Magnetic Fields	169
12.4	One Major Assumption	170
12.5	Exercises	172
13	Electromagnetic Radiation	173
13.1	The Electromagnetic Spectrum	173
13.2	Electromagnetic Intensity	174
13.3	The Poynting Vector	176
13.4	Radiation Pressure	177
13.4.1	The Momentum-Energy Relation for Electromagnetic Waves	177
13.4.2	Example # 1: The Solar Sail	178
13.5	Polarization of Light	180
13.5.1	Polarization by Transmission	180
13.5.2	Example # 1: Polarizing Sunglasses	181
13.5.3	Example # 2: The Sky	182
13.5.4	Polarization by Reflection	182
13.5.5	Example # 3: Sheet Glass Reflection	183
13.6	Exercises	184
14	Diffraction and Interference of Light	187
14.1	The Concept of Diffraction	187
14.2	Interference of Coherent Waves is Caused by Phase Difference	188
14.2.1	Phase Difference as a Path Length Difference	188
14.2.2	Phase Difference Due to Reflection	189
14.3	Thin-Film Interference	189
14.3.1	Example # 1: A Thin Film of Water, Between Air and Glass	190
14.3.2	Example # 2: An Oil Film on Water	191
14.3.3	Example # 3: A Thin Film of Air, Between Glass Plates	192
14.3.4	Example # 4: A Wedge of Air, Between Glass Plates	193
14.3.5	Example # 5: Newton's Rings	193
14.3.6	Example # 6: Soap Bubbles	194
14.3.7	Example # 7: Anti-Reflective (AR) Coatings	195
14.3.8	Example # 8: Interference Filters	195
14.4	The Michelson Interferometer	196
14.5	Two-Slit Interference	197
14.6	Single-Slit Diffraction	198
14.7	Superposition of Slit Interference and Diffraction Patterns	199
14.8	Diffraction Gratings	200
14.8.1	Spectral Resolving Power	201
14.9	Phasors and Phase Difference	202
14.9.1	Drawing the Superposition of Two Waves	202

14.9.2	Finding the Phase Difference	203
14.9.3	Example # 1: Finding the Phase Difference for 2-Slit Interference	203
14.9.4	Example # 2: Maximum and Minimum Amplitudes for 3-Slit Interference	205
14.10	Interference Pattern Intensity	207
14.10.1	Intensity Derived for 2-Slit Interference	207
14.10.2	Intensity for Single-Slit Diffraction	208
14.10.3	The Superposition of Single- and Two-Slit Intensities	209
14.11	Two Types of Diffraction	210
14.11.1	Fraunhofer Diffraction: Distant Sources of Light	210
14.11.2	Fresnel Diffraction: Near Sources of Light	210
14.11.3	Example Photographs of Fresnel Diffraction	211
14.12	Exercises	212
15	Wave-Particle Duality	215
15.1	Light as a Particle: The Photon	215
15.2	Energy Levels in an Atom: Hydrogen	216
15.3	Light-Atom Interactions	218
15.3.1	Elastic Scattering	218
15.3.2	Inelastic Scattering	218
15.3.3	Atomic Emission	218
15.3.4	Compton Scattering	220
15.4	Example # 1: The Photoelectric Effect	221
15.5	Example # 2: The LASER	222
15.6	Other Types of Lasers	223
15.7	Exercises	225
IV	Laboratory Experiments	227
16	Laboratory # 1: Shadows	229
V	Appendices	243
A	Statistics	245
A.1	Systematic and Random Errors	245
A.2	Propagating Uncertainties	246
A.2.1	Sums and Differences	246
A.2.2	The General Method	247
A.2.3	Weighted Sums and Differences	248
A.2.4	Products and Quotients	248
A.3	Least-Squares Fit to a Linear Trend in Data	249
B	Complex Notation	251

CONTENTS

9

C References

253

Part I

Geometric Optics

Chapter 1

The Rectilinear Propagation of Light

1.1 A Demonstration of Shadows

We have a bright lamp, with an object that casts a shadow on the screen. A representation of the setup from class is drawn in Figure 1.1.

The lamp and the object can both be moved. How can they be moved in order to make the shadow grow larger? How can they be moved in order to make the shadow grow smaller? Why is the edge of the shadow indistinct, and not sharp?

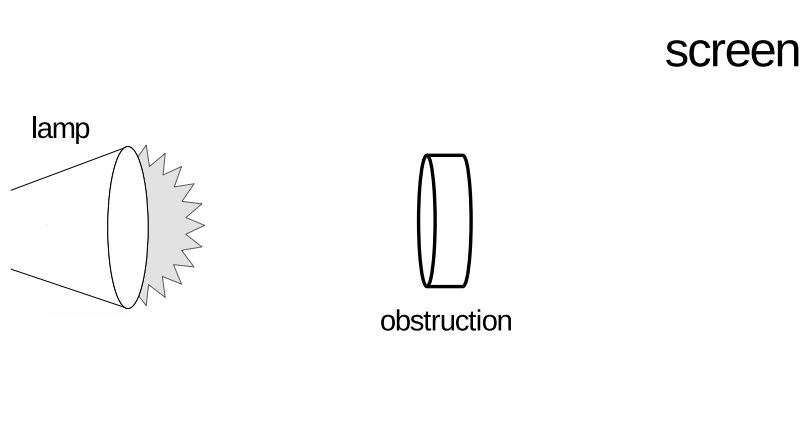


Figure 1.1: Demonstration of a shadow being cast by an object in front of a lamp.

1.2 Huygens' Principle

To understand how shadows are made, we must first understand how light propagates, *i.e.*, moves through empty space when there are no objects to obstruct it. We begin by discussing wave fronts.

1.2.1 Wave fronts

Let us draw a wave front propagating from a light bulb in Figure 1.2. Every point on this wave front is the source of an infinitesimal wavelet, which propagates outwards from its source on the wave front. When all of these little wavelets are added together, they form a new wave front. This continues so that all of the wave fronts form a series of embedded arcs around the light bulb.

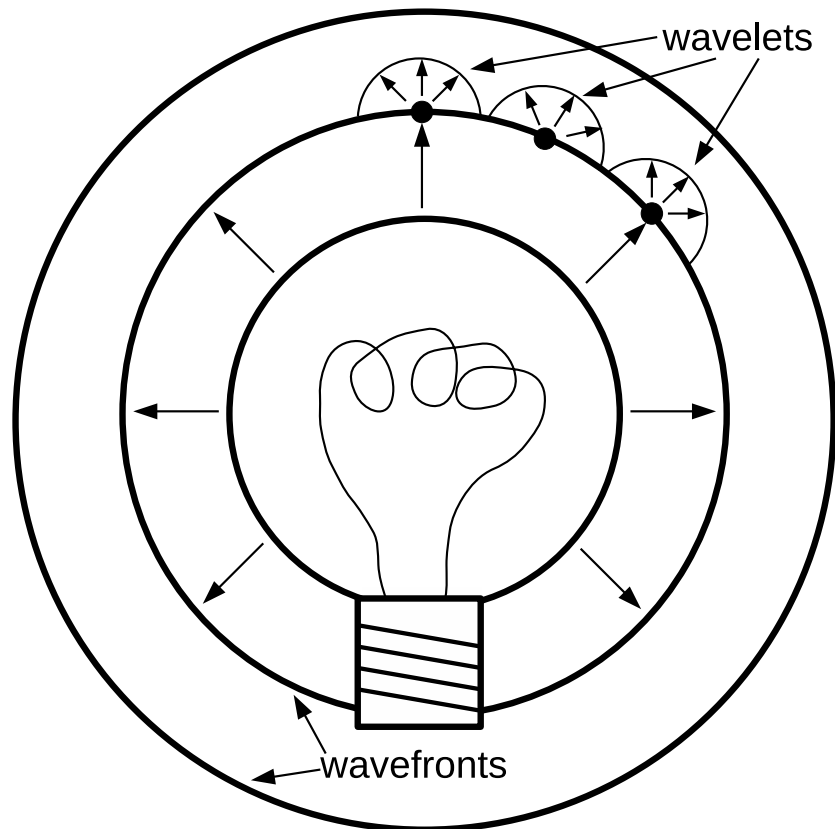


Figure 1.2: Wave fronts propagating from a light bulb.

1.2.2 The Law of Rectilinear Propagation

Follow a path from the light bulb through each wave front, connecting similar wavelets in each wave front, and we produce straight lines, which we can illustrate in Figure 1.3. These straight lines are called **light rays**. The fact that light will follow a straight path, in the absence of anything getting in the way, is called the law of rectilinear propagation (meaning that light moves in straight paths).

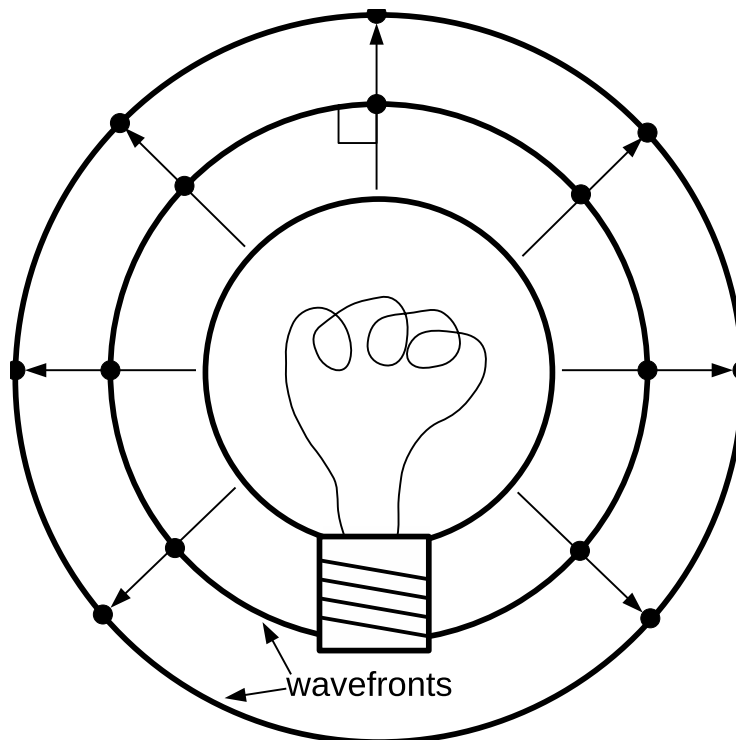


Figure 1.3: The formation of light rays.

Light rays have a couple of geometric properties. The first and most obvious is that they are orthogonal to the wave fronts and therefore indicate the directions in which the wave front is propagating. In the case of the light bulb, the light rays appear radial and point away from the light bulb in all directions.

The second and more subtle property is that, being straight, then in the Cartesian coordinate system in which the light bulb exists the light rays follow the path of shortest time between two points. We'll get back to this property in Chapter 2.

1.2.3 Physical Evidence

How do we know that light rays offer a true description of how light actually moves? We can observe the rectilinear propagation of light in nature, as illustrated in Figure 1.4. Light from the Sun propagates through a forest, blocked by branches and trunks, forming narrow bands that always follow straight paths. This image illustrates that the law of rectilinear propagation is an apt description of how light moves.



Figure 1.4: Sunlight shining through a forest. Image credit: Niki Pike, https://nikipike.com/wp-content/uploads/2016/02/IMG_7708.jpg

1.2.4 Light Rays and the Anatomy of a Shadow

Now that we have an idea for how light propagates, we can use light rays to draw shadows cast by an object and then use those light rays to relate the object and light source distances to the size of the shadow. Figure 1.5 on the next page sketches the shadow cast in our demonstration (Fig. 1.1) in detail. The variables we will use are:

- S : The height of the light source
- O : The height of the object
- U : The height of the umbra
- P : The height of the penumbra (partial shadow)

- x : The distance from light source to object
- L : The distance from light source to screen

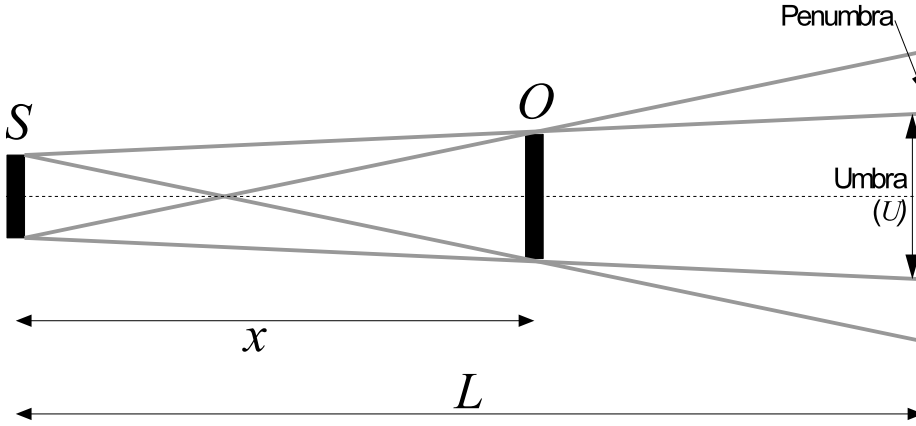


Figure 1.5: Relating source and object distances to the size and appearance of the shadow cast.

We will make measurements from the center line, so that $S/2$ represents the height of the light source above the center line, and so forth.

The technique we are going to employ relies on finding similar triangles, *i.e.* those triangles that share the same angles, even if their sides are different lengths. For example, the height of the object *above* the light source appears to be proportional to the height of the umbra above the light source. We can therefore equate the ratio of the vertical height and horizontal length for each of those triangles, since that ratio equals the tangent of the apex angle of the two similar triangles:

$$\frac{O/2 - S/2}{x} = \frac{U/2 - S/2}{L} \quad (1.1)$$

Carrying down the 2, this becomes

$$\frac{O - S}{2x} = \frac{U - S}{2L} \quad (1.2)$$

This equation can be simplified using cross multiplication and removing a common factor from both sides:

$$(O - S)L = (U - S)x \quad (1.3)$$

Since we wish to discover a relationship between shadow size and object/light source distances, we should now solve for U in the above equation.

$$U = \frac{OL - SL + Sx}{x} \quad (1.4)$$

We will rearrange this slightly so that L , x , and $L - x$ all appear separately:

$$U = \frac{OL - S(L - x)}{x} \quad (1.5)$$

so that now we can use this equation to show how each of the distances are responsible for the size of the umbra. More than any other single variable, x decides the size of the umbra. U and x are inversely proportional, so that if the object is very close to the light source, then the umbra is very large. There is a second important relationship that is easy to observe: as x approaches L , then U approaches O .

Now let us take a look at the size of the penumbra, which is oftentimes harder to measure than the umbra because it is not as dark. We will use the light ray starting from the bottom of the source and leading all the way to the top of the penumbra. This can be difficult to see, so take time discerning the following similar triangles equation:

$$\frac{O - (O - S)/2}{x} = \frac{P/2 + S/2}{L} \quad (1.6)$$

We will carry down the two as before, plus check our algebra to ensure that

$$\frac{O + S}{2x} = \frac{P + S}{2L} \quad (1.7)$$

Then we cross-multiply and remove the common factor from both sides to get

$$(O + S)L = (P + S)x \quad (1.8)$$

Solving for P , the size of the penumbra, we find

$$P = \frac{OL + SL - Sx}{x} \quad (1.9)$$

which can be re-written as

$$P = \frac{OL + S(L - x)}{x} \quad (1.10)$$

That looks pretty similar to our equation for the umbra, except that the two terms in the numerator are added instead of subtracted. In essence, however, what this shows is that the size of the penumbra has the same basic dependencies as does the size of the umbra: It becomes larger when x is smaller, and in the limiting case of L and x being similar then P becomes the size of the object itself.

Of course, what we actually see of the penumbra is an annulus around the umbra. How thick that annulus appears can be shown by subtracting our equation for the umbra from our equation for the penumbra.

$$P - U = \frac{OL + S(L - x)}{x} - \frac{OL - S(L - x)}{x} = 2S \frac{L - x}{x} \quad (1.11)$$

Then, the thickness of the penumbral annulus can be written by dividing both sides of the above equation by 2:

$$\frac{P - U}{2} = S \frac{L - x}{x} \quad (1.12)$$

The penumbra will appear large when the source is physically large. The size of the penumbra can be further enhanced if the distance from object to screen is large compared to the distance between the object and light source.

We will explore these relationships more in this chapter's laboratory: *Using Shadows to Measure the Sizes of the Earth, Sun, and Moon.*